Summer School: Global Sensivity Analysis and Poincaré inequalities

6-8 July 2022

Institut de Mathématiques de Toulouse
Amphi Schwartz

Schedule

	Wednesday 6	**Thursday 7**	**Friday 8**
8:45-9:00	Opening		
9:00-10:30	Clémentine Prieur 1	Michel Bonnefont 1	Clémentine Prieur 3
10:30-11:00	Coffee	Coffee	Coffee
11:00-12:30	Olivier Roustant 1	Clémentine Prieur 2	Michel Bonnefont 2
12:30-14:00	Lunch	Lunch	Lunch
14:00-14:40	Clément Benesse	Ronan Herry	
14:40-15:20	Lucas Journel	Gabriel Sarazin	PhD defense Clément Steiner
15:20-15:50	Coffee	Coffee	
15:50-16:30	Julien Demange-Chryst	Jordan Serres	
16:30-17:10	Pierre Le Bris	Anouar Meynaoui	
17:10-19:30			
19:30-23:00		Dinner	

Mini-courses

Clémentine Prieur - Université Grenoble Alpes

Global sensitivity analysis and dimension reduction.
Lecture 1. Variance-based sensitivity analysis. I will present the general framework of variance-based sensitivity analysis, from the definition of sensitivity indices to the presentation of different inference strategies.

Lecture 2. Sensitivity analysis and dimension reduction. I will make a link between between sensitivity analysis and dimension reduction via the so-called active subspace methodology, with an extension to the framework of vectorial output. Finally, I will present a methodology we introduced recently for nonlinear dimension reduction.
Lecture 3. From variance-based to more general sensitivity indices. I will present recent results on sensitivity analysis targeted to the analysis of models with outputs valued in general metric spaces. I will also discuss sensitivity analysis in the framework of dependent inputs.

Olivier Roustant - Institut de Mathématiques de Toulouse

Poincaré inequalities and application to sensivitity analysis. Part I: the one-dimensional case.
This session brings theory on 1-dimensional Poincaré inequalities with a focus on computational aspects. We show how to compute the (optimal) Poincaré constant from a spectral problem, either explicitly in favorable cases, or numerically in general. We also give a characterization of the Poincaré basis formed by the eigenfunctions associated to the Poincaré inequalities. Finally, we illustrate the strength of the Poincaré inequalities framework for doing a global sensitivity analysis of a multivariate function when derivatives are available.

Michel Bonnefont - Institut de Mathématiques de Bordeaux

Poincaré inequalities and application to sensivitity analysis. Part II: the high-dimensional setting.
The goal of this course will be to survey some of the methods that provide the Poincaré inequality with an explicit constant in dimension $d \geq 1$.

Talks

Clément Benesse - Institut de Mathématiques de Toulouse

When Global Sensitivity Analysis provides insight into Group Fairness.
In recent years, Algorithmic Fairness has become a widely studied subject in Machine Learning, mainly because of societal needs. We draw theoretical links between this field and Sensitivity Analysis. This allows us to translate various results found in the Global Sensitivity Analysis literature into the Fairness framework, providing insight for future implementation in the civil society.

Julien Demange-Chryst - Institut de Mathématiques de Toulouse

Shapley effect estimation in reliability-oriented sensitivity analysis with dependent inputs by importance sampling.
Reliability-oriented sensitivity analysis aims at combining both reliability and sensitivity analyses by quantifying the influence of each input variable of a numerical model on a quantity of interest related to its failure. In particular, target sensitivity analysis focuses on the occurrence of the failure, and more precisely aims to determine which inputs are more likely to lead to the failure of the system. The Shapley effects are quantitative global sensitivity indices which are able to deal with dependent input variables. They have

been recently adapted to the target sensitivity analysis framework. In this contribution, we investigate importance-sampling-based estimation schemes of these indices which are more efficient than the existing ones when the failure probability is small. Moreover, an extension to the case where only an i.i.d. input/output N-sample distributed according to the importance sampling auxiliary distribution is available is proposed. This extension allows to estimate the Shapley effects only with a data set distributed according to the importance sampling auxiliary distribution stemming from a reliability analysis without additional calls to the numerical model. In addition, we study theoretically the absence of bias of some estimators as well as the benefit of importance sampling, and finally, realistic test cases show the practical interest of the proposed methods.

Ronan Herry - Bonn University

Transport inequalities for point processes.
Transport inequalities were introduced in the 90's by K. Marton and M. Talagrand in order to derive dimension-free concentration of measures results. In its simplest form, a transport inequality estimates from above the distance between two probability measures and the sum of their entropy relatively to some reference measures. For instance, the celebrated inequality by Talagrand links the Wasserstein 2 transport distances to the entropy relatively to the Gaussian distribution and gives back Gaussian concentration of measures.
I will take the opportunity of this talk to review the ideas of Talagrand and Marton, and present recent developments in this field. In particular, I will focus:

1) on the umbrella concept of generalized transport inequalities by Gozlan, Roberto, Tetali and Samson subsuming all previous instances of transport inequalities.

2) on concrete examples of transport inequalities that can be derived for point processes (that is, the law of random points), and in particular for Poisson point process. Based on a joint work with Nathael Gozlan and Giovanni Peccati.

Lucas Journel - Sorbonne Université

Convergence of the kinetic annealing for general potentials.
The goal of a simulated annealing is to find, via a stochastic process, the minimum of some function $U : \mathbb{R}^d \mapsto \mathbb{R}_+$. To this end we study the process:

$$\begin{cases} \mathrm{d}X_t = Y_t \mathrm{d}t \\ \mathrm{d}Y_t = -\nabla U(X_t)\mathrm{d}t - \gamma_t Y_t \mathrm{d}t + \sqrt{2\gamma_t \beta_t^{-1}}\mathrm{d}B_t\,, \end{cases} \tag{1}$$

where $\beta_t = \frac{\ln(e^{c\beta_0}+t)}{c}$. Let c^* be the largest energy barrier of U. We proved under mild assumptions on the potential U the convergence of the kinetic annealing towards the minimum of U for $c > c^*$, as well as the non-convergence for $c < c^*$.

Pierre Le Bris - Sorbonne Université

Uniform in time propagation of chaos for the 2D vortex model.
In this talk, after introducing the general context of propagation of chaos phenomenon, we show how we may obtain uniform in time propagation of chaos for a class of singular interaction kernels, extending the results of Fournier-Hauray-Mischler (JEMS) and Jabin-Wang (Inv. Math.). In particular, our models contain the Biot-Savart kernel on the

torus and thus the 2D vortex model. The strategy is to combine the relative entropy approach of Jabin-Wang with functional inequalities as well as uniform bounds on all the derivatives of the solution of the non linear limit equation, in order to control both the entropy dissipation and the constants appearing in the large deviation estimates.
This is joint work with Arnaud Guillin (LMBP, Université Clermont-Auvergne) and Pierre Monmarché (LJLL, Sorbonne Université).

Anouar Meynaoui - Université Grenoble Alpes

Second-level global sensitivity analysis of numerical simulators.
To perform Global sensitivity analysis (GSA), statistical tools based on inputs/output dependence measures are commonly used. We focus here on the Hilbert-Schmidt Independence Criterion (HSIC). Sometimes, the probability distributions modeling the uncertainty of inputs may be themselves uncertain and it is important to quantify their impact on GSA results. We call it here the second-level global sensitivity analysis (GSA2). However, GSA2, when performed with a Monte Carlo double-loop, requires a large number of model evaluations which is intractable with CPU time expensive simulators. To cope with this limitation, we propose a new statistical methodology based on a Monte Carlo single-loop with a limited calculation budget.

Gabriel Sarazin - CEA

What is hidden behind the Sobolev kernels involved in the HSIC-ANOVA decomposition?
In many industrial fields, the numerical simulators that model physical phenomena are very expensive from a computational viewpoint. In this context, only a very limited number of computer experiments can be planned. Statistical learning techniques helps build a surrogate model but this task becomes excessively challenging when the simulation code takes a high number (dozens, if not hundreds) of uncertain parameters as input variables. To escape from this deadlock, global sensitivity analysis (GSA) may be used as a tool for dimension reduction. In particular, the sensitivity measures which are based on the Hilbert-Schmidt independence criterion (HSIC) have been increasingly used over the past few years because they pave the way to several variable selection procedures. However, the so-called HSIC indices still suffer from their lack of interpretability, especially among non-specialists who rather ask for percentage-like indicators. To ease interpretation, HSIC-ANOVA indices have been recently introduced to recover a strict separation of main effects and interactions. This breakthrough was obtained after assuming mutual independence among input variables and provided that ANOVA kernels, such as Sobolev kernels, are used to estimate HSIC-ANOVA indices. In the literature, very little has been said about Sobolev kernels. In this talk, their feature maps are investigated in three different ways. Firstly, a simulation-based approach named kernel feature analysis (KFA) allows to distinguish different behaviors depending on the integer value given to their smoothness parameter. Secondly, the eigenvalue problem that goes with their Mercer decomposition is transformed into a Cauchy problem that can be solved in one specific case. Thirdly, a series expansion of their translation-invariant term discloses a fully analytical characterization of their reproducing kernel Hilbert spaces (RKHS). Based on those theoretical results, it can be proved that all Sobolev kernels are characteristic, which means HSIC-ANOVA indices are well-adapted association measures to detect independence among input-output pairs.

Jordan Serres - Institut de Mathématiques de Toulouse

Stability of the Poincaré constant.
We will present a study of the stability of the sharp Poincaré constant of the invariant probability measure of a reversible diffusion process. We will discuss the spectral interpretation of Poincaré inequalities and Stein's method, which are the main ingredients of the proof. In particular, these results will be applied to gamma distributions, beta distributions, and one-dimensional log-concave measures.

Clément Steiner - Institut de Mathématiques de Toulouse

PhD defense: *On the use of intertwining relations in the study of self-adjoint Markov generators. Application to functional and spectral inequalities and sensitivity analysis.*
This thesis is part of a series of works carried out by Aldéric Joulin, Michel Bonnefont et alius, which aims at using intertwining relations to infer properties of some Markov generators. The present work deals specifically with three properties: Poincaré inequalities, logarithmic Sobolev inequalities and spectral estimates. Both above inequalities are widely use tools in infinite-dimensional analysis, that relate to the latter generators and underlying Boltzmann-Gibbs invariant distribution. In the first chapter, a method based on Feynman-Kac semigroups is proposed to infer new estimates, in relation to the logarithmic Sobolev inequality. The connexion between generators and stochastic processes is explored via a representation theorem for Feynman-Kac semigroups. In the second chapter, an algebraic approach to the estimation of eigenvalues of the aforementioned generators is discussed. This work echoes a related recent article by Emanuel Milman, in which he used optimal transport results in this very purpose. Multiplicities are addressed as well, in relation to the recent work of Franck Barthe and Boaz Klartag. In the last chapter, the relation between Poincaré inequalities and sensitivity analysis is investigated, particularly in order to compare two types of sensitivity indices. An estimation method related to this inequality is developed in dimension two, using finite elements methods.

Organizer: Aldéric Joulin - Institut de Mathématiques de Toulouse

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